CHAPTER 8
LOGARITHMS AND THE SLIDE RULE

Logarithms represent a specialized use of exponents. By means of logarithms, computation with large masses of data can be greatly simplified. For example, when logarithms are used, the process of multiplication is replaced by simple addition and division is replaced by subtraction. Raising to a power by means of logarithms is done in a single multiplication, and extracting a root reduces to simple division.

DEFINITIONS

In the expression $2^3 = 8$, the number 2 is the base (not to be confused with the base of the number system), and 3 is the exponent which must be used with the base to produce the number 8. The exponent 3 is the logarithm of 8 when the base is 2. This relationship is usually stated as follows: The logarithm of 8 to the base 2 is 3. In general, the logarithm of a number N with respect to a given base is the exponent which must be used with the base to produce N. Table 8-1 illustrates this.

Table 8-1.—Logarithms with various bases.

Exponential form	Logarithmic form
$2^3 = 8$	$\log_2 8 = 3$
$4^2 = 16$	$\log_4 16 = 2$
$5^0 = 1$	$\log_5 1 = 0$
$27^{2/3} = 9$	$\log_{27} 9 = 2/3$

Table 8-1 shows that the logarithmic relationship may be expressed equally well in either of two forms; these are the exponential form and the logarithmic form. Observe, in table 8-1, that the base of a logarithmic expression is indicated by placing a subscript just below and to the right of the abbreviation "log." Observe also that the word "logarithm" is abbreviated without using a period.

The equivalency of the logarithmic and exponential forms may be used to restate the fundamental definition of logarithms in its most useful form, as follows:

$$b^x = N \text{ implies that } \log_b N = x$$

In words, this definition is stated as follows: If the base b raised to the x power equals N, then x is the logarithm of the number N to the base b.

One of the many uses of logarithms may be shown by an example in which the base is 2. Table 8-2 shows the powers of 2 from 0 through 20. Suppose that we wish to use logarithms to multiply the numbers 512 and 256, as follows:

From table 8-2, $512 = 2^9$

$256 = 2^8$

Then $512 \times 256 = 2^9 \times 2^8$

$= 2^{17}$

and from the table again $2^{17} = 131072$

It is seen that the problem of multiplication is reduced to the simple addition of the exponents 9 and 8 and finding the corresponding power in the table.

Table 8-2 (A) shows the base 2 in the exponential form with its corresponding powers. The actual computation in logarithmic work does not require that we record the exponential form. All that is required is that we add the appropriate exponents and have available a table in which we can look up the number corresponding to the new exponent after adding. Therefore, table 8-2 (B) is adequate for our purpose. Solving the foregoing example by this table, we have the following:

$$\log_2 512 = 9$$

$$\log_2 256 = 8$$

$$\log_2 \text{ of the product} = 17$$

Therefore, the number we seek is the one in the table whose logarithm is 17. This number is 131,072. In this example, we found the exponents directly, added them since this was a

Table 8-2.—Exponential and logarithmic tables for the base 2.

(A) Powers of 2 from 0 through 20	(B) Logarithms for the base 2 and corresponding powers	
	Log	Number
$2^0 = 1$	0	1
$2^1 = 2$	1	2
$2^2 = 4$	2	4
$2^3 = 8$	3	8
$2^4 = 16$	4	16
$2^5 = 32$	5	32
$2^6 = 64$	6	64
$2^7 = 128$	7	128
$2^8 = 256$	8	256
$2^9 = 512$	9	512
$2^{10} = 1024$	10	1024
$2^{11} = 2048$	11	2048
$2^{12} = 4096$	12	4096
$2^{13} = 8192$	13	8192
$2^{14} = 16384$	14	16384
$2^{15} = 32768$	15	32768
$2^{16} = 65536$	16	65536
$2^{17} = 131072$	17	131072
$2^{18} = 262144$	18	262144
$2^{19} = 524288$	19	524288
$2^{20} = 1048576$	20	1048576

multiplication problem, and located the corresponding power. This avoided the unnecessary step of writing the base 2 each time.

Practice problems. Use the logarithms in table 8-2 to perform the following multiplication:

1. 64 x 128
2. 1,024 x 256
3. 128 x 4,096
4. 512 x 2,048

Answers:

1. 8,192
2. 262,144
3. 524,288
4. 1,048,576

NATURAL AND COMMON LOGARITHMS

Many natural phenomena, such as rates of growth and decay, are most easily described in terms of logarithmic or exponential formulas. Furthermore, the geometric patterns in which certain seeds grow (for example, sunflower seeds) is a logarithmic spiral. These facts explain the name "natural logarithms." Natural logarithms use the base e, which is an irrational number approximately equal to 2.71828. This system is sometimes called the Napierian system of logarithms, in honor of John Napier, who is credited with the invention of logarithms.

To distinguish natural logarithms from other logarithmic systems the abbreviation, ln, is sometimes used. When ln appears, the base is understood to be e and need not be shown. For example, either $\log_e 45$ or ln 45 signifies the natural logarithm of 45.

COMMON LOGARITHMS

As has been shown in preceding paragraphs, any number may be used as a base for a system of logarithms. The selection of a base is a matter of convenience. Briggs in 1617 found that base 10 possessed many advantages not obtainable in ordinary calculations with other bases. The selection of 10 as a base proved so satisfactory that today it is used almost exclusively for ordinary calculations. Logarithms with 10 as a base are therefore called COMMON LOGARITHMS.

When 10 is used as a base, it is not necessary to indicate it in writing logarithms. For example,

$$\log 100 = 2$$

is understood to mean the same as

$$\log_{10} 100 = 2$$

If the base is other than 10, it must be specified by the use of a subscript to the right and below the abbreviation "log." As noted in the foregoing discussion of natural logarithms, the use of the distinctive abbreviation "ln" eliminates the need for a subscript when the base is e.

It is relatively easy to convert common logarithms to natural logarithms or vice versa, if necessary. It should be noted further that each system has its peculiar advantages, but for most everyday work, the common system is more often used. A simple relation connects the two systems. If the common logarithm of a number can be found, multiplying by 2.3026 gives the natural logarithm of the number. For example,

$$\log 1.60 = 0.2041$$

$$\ln 1.60 = 2.3026 \times 0.2041 = 0.4700$$

Thus the natural logarithm of 1.60 is 0.4700, correct to four significant digits.

Conversely, multiplying the natural logarithm by 0.4343 gives the common logarithm of a number. As might be expected, the conversion factor 0.4343 is the reciprocal of 2.3026. This is shown as follows:

$$\frac{1}{2.3026} = 0.4343$$

Positive Integral Logarithms

The derivation of positive whole logarithms is readily apparent. For example, we see in table 8-3 (B) that the logarithm of 10 is 1. The number 1 is simply the exponent of the base 10 which yields 10. This is shown in table 8-3 (A) opposite the logarithmic equation. Similarly,

$10^0 = 1$ . . . . $\log 1 = 0$

$10^2 = 100$ . . . . $\log 100 = 2$

$10^3 = 1{,}000$ . . . . . . $\log 1{,}000 = 3$

$10^4 = 10{,}000$ . . . . . $\log 10{,}000 = 4$

Table 8-3.—Exponential and corresponding logarithmic notations using base 10.

A.			B.	
10^{-4}	$= \frac{1}{10^4}$	$= 0.0001$	$\log 0.0001$	$= -4$
10^{-3}	$= \frac{1}{10^3}$	$= 0.001$	$\log 0.001$	$= -3$
10^{-2}	$= \frac{1}{10^2}$	$= 0.01$	$\log 0.01$	$= -2$
10^{-1}	$= \frac{1}{10}$	$= 0.1$	$\log 0.1$	$= -1$
$10^{-1/2}$	$= \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$	$= 0.31623$	$\log 0.31623$	$= -0.5$ $= 0.5 - 1$
10^0		$= 1$	$\log 1$	$= 0$
$10^{1/2}$	$= \sqrt{10}$	$= 3.1623$	$\log 3.1623$	$= 0.5$
10^1		$= 10$	$\log 10$	$= 1$
$10^{3/2}$	$= 10\sqrt{10}$	$= 31.623$	$\log 31.623$	$= 1.5$
10^2		$= 100$	$\log 100$	$= 2$
$10^{5/2}$	$= 10^2(\sqrt{10})$	$= 316.23$	$\log 316.23$	$= 2.5$
10^3		$= 1{,}000$	$\log 1{,}000$	$= 3$
$10^{7/2}$	$= 10^3(\sqrt{10})$	$= 3162.3$	$\log 3162.3$	$= 3.5$
10^4		$= 10{,}000$	$\log 10{,}000$	$= 4$

Positive Fractional Logarithms

Referring to table 8-3, notice that the logarithm of 1 is 0 and the logarithm of 10 is 1. Therefore, the logarithm of a number between 1 and 10 is between 0 and 1. An easy way to verify this is to consider some numbers between 1 and 10 which are powers of 10; the exponent in each case will then be the logarithm we seek. Of course, the only powers of 10 which produce numbers between 1 and 10 are fractional powers.

EXAMPLE: $10^{1/2} = 3.1623$ (approximately)

$$10^{0.5} = 3.1623$$

Therefore, log 3.1623 = 0.5

Other examples are shown in the table for $10^{3/2}$, $10^{5/2}$, and $10^{7/2}$. Notice that the number that represents $10^{3/2}$, 31.623, logically enough lies between the numbers representing 10^1 and 10^2—that is, between 10 and 100. Notice also that $10^{5/2}$ appears between 10^2 and 10^3, and $10^{7/2}$ lies between 10^3 and 10^4.

Negative Logarithms

Table 8-3 shows that negative powers of 10 may be fitted into the system of logarithms. We recall that 10^{-1} means $\frac{1}{10}$, or the decimal fraction, 0.1. What is the logarithm of 0.1?

SOLUTION: $10^{-1} = 0.1$; log 0.1 = -1

Likewise $10^{-2} = 0.01$; log 0.01 = -2

Negative Fractional Logarithms

Notice in table 8-3 that negative fractional exponents present no new problem in logarithmic notation. For example, $10^{-1/2}$ means $\frac{1}{\sqrt{10}}$.

$$\frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} = 0.31623$$

What is the logarithm of 0.31623?

SOLUTION:

$$10^{-1/2} = 0.31623; \log 0.31623 = -\frac{1}{2} = -0.5$$

Table 8-3 shows logarithms for numbers ranging from 0.0001 to 10,000. Notice that there are only 8 integral logarithms in the entire range. Excluding zero logarithms, the logarithms for all other numbers in the range are fractional or contain a fractional part. By the year 1628, logarithms for all integers from 1 to 100,000 had been computed. Practically all of these logarithms contain a fractional part. It should be remembered that finding the logarithm of a number is nothing more than expressing the number as a power of 10. Table 8-4 shows the numbers 1 through 10 expressed as powers of 10. Most of the exponents which comprise logarithms are found by methods beyond the scope of this text. However, it is not necessary to know the process used to obtain logarithms in order to make use of them.

Table 8-4.—The numbers 1 through 10 expressed as powers of 10.

$1 = 10^0$	$6 = 10^{0.77815}$
$2 = 10^{0.30103}$	$7 = 10^{0.84510}$
$3 = 10^{0.47712}$	$8 = 10^{0.90309}$
$4 = 10^{0.60206}$	$9 = 10^{0.95424}$
$5 = 10^{0.69897}$	$10 = 10^1$

COMPONENTS OF LOGARITHMS

The fractional part of a logarithm is usually written as a decimal. The whole number part of a logarithm and the decimal part have been given separate names because each plays a special part in relation to the number which the logarithm represents. The whole number part of a logarithm is called the CHARACTERISTIC. This part of the logarithm shows the position of the decimal point in the associated number. The decimal part of a logarithm is called the MANTISSA.

For a particular sequence of digits making up a number, the mantissa of a common logarithm is always the same regardless of the position of the decimal point in that number. For example, log 5270 = 3.72181; the mantissa is 0.72181 and the characteristic is 3.

CHARACTERISTIC

The characteristic of a common logarithm shows the position of the decimal point in the associated number. The characteristic for a given number may be determined by inspection. It will be remembered that a common logarithm is simply an exponent of the base 10. It is the power of 10 when a number is written in scientific notation.

When we write log 360 = 2.55630, we understand this to mean $10^{2.55630} = 360$. We know that the number is 360 and not 36 or 3,600 because the characteristic is 2. We know 10^1 is 10, 10^2 is 100, and 10^3 is 1,000. Therefore, the number whose value is $10^{2.55630}$ must lie between 100 and 1,000 and of course any number in that range has 3 digits.

Suppose the characteristic had been 1: where would the decimal point in the number be placed? Since 10^1 is 10 and 10^2 is 100, any number whose logarithm is between 1 and 2 must lie between 10 and 100 and will have 2 digits. Notice how the position of the decimal point changes with the value of the characteristic in the following examples:

$$\log 36{,}000 = 4.55630$$
$$\log 3{,}600 = 3.55630$$
$$\log 360 = 2.55630$$
$$\log 36 = 1.55630$$
$$\log 3.6 = 0.55630$$

Note that it is only the characteristic that changes when the decimal point is moved. An advantage of using the base 10 is thus revealed: If the characteristic is known, the decimal point may easily be placed. If the number is known, the characteristic may be determined by inspection; that is, by observing the location of the decimal point.

Although an understanding of the relation of the characteristic to the powers of 10 is necessary for thorough comprehension of logarithms, the characteristic may be determined mechanically by application of the following rules:

1. For a number greater than 1, the characteristic is positive and is one less than the number of digits to the left of the decimal point in the number.

2. For a positive number less than 1, the characteristic is negative and has an absolute value one more than the number of zeros between the decimal point and the first nonzero digit of the number.

Table 8-5 contains examples of each type of characteristic.

Practice problems. In problems 1 through 4, write the characteristic of the logarithm for each number. In 5 through 8, place the decimal

Table 8-5.—Positive and negative characteristics.

Number	Power of 10	Digits in number to the left of decimal point	Characteristic
	Between:		
134	10^2 and 10^3	3	2
13.4	10^1 and 10^2	2	1
1.34	10^0 and 10^1	1	0
		Zeros between decimal point and first nonzero digit	
0.134	10^{-1} and 10^0	0	-1
0.0134	10^{-2} and 10^{-1}	1	-2
0.00134	10^{-3} and 10^{-2}	2	-3

point in each number as indicated by the characteristic (c) given for each.

1. 4,321 2. 1.23 3. 0.05 4. 12
5. 123; c = 4 6. 8,210; c = 0
7. 8; c = -1 8. 321; c = -2

Answers:

1. 3 2. 0 3. -2 4. 1
5. 12,300 6. 8.210 7. 0.8 8. 0.0321

Negative Characteristics

When a characteristic is negative, such as -2, we do not carry out the subtraction, since this would involve a negative mantissa. There are several ways of indicating a negative characteristic. Mantissas as presented in the table in the appendix are always positive and the sign of the characteristic is indicated separately. For example, where log 0.023 = $\bar{2}.36173$, the bar over the 2 indicates that only the characteristic is negative—that is, the logarithm is -2 + 0.36173.

Another way to show the negative characteristic is to place it after the mantissa. In this case we write 0.36173-2.

A third method, which is used where possible throughout this chapter, is to add a certain quantity to the characteristic and to subtract the same quantity to the right of the mantissa. In the case of the example, we may write:

$$\begin{array}{l} \bar{2}.36173 \\ \underline{10 \qquad -10} \\ 8.36173-10 \end{array}$$

In this way the value of the logarithm remains the same but we now have a positive characteristic as well as a positive mantissa.

MANTISSA

The mantissa is the decimal part of a logarithm. Tables of logarithms usually contain only mantissas since the characteristic can be readily determined as explained previously. Table 8-6 shows the characteristic, mantissa, and logarithm for several positions of the decimal point using the sequence of digits 4, 5, 6. It will be noted that the mantissa remains the same for that particular sequence of digits, regardless of the position of the decimal point.

Table 8-6.—Effect of changes in the location of the decimal point.

Number	Characteristic	Mantissa	Logarithm
45,600	4	0.6590	4.6590
4,560	3	0.6590	3.6590
456	2	0.6590	2.6590
45.6	1	0.6590	1.6590
4.56	0	0.6590	0.6590
0.456	-1	0.6590	0.6590-1
0.0456	-2	0.6590	0.6590-2
0.00456	-3	0.6590	0.6590-3

Appendix I of this training course is a table which includes the logarithms of numbers from 1 to 100. For our present purpose in using this table, we are concerned only with the first and sixth columns.

The first column contains the number and the sixth column contains its logarithm. For example, if it is desired to find the logarithm of 45, we would find the number 45 in the first column, look horizontally across the page to column 6 and read the logarithm, 1.65321. A glance down the logarithm column will reveal that the logarithms increase in value as the numbers increase in value.

It must be noted in this particular table that both the mantissa and the characteristic are given for the number in the first column. This is simply an additional aid, since the characteristic can easily be determined by inspection.

Suppose that we wish to use the table of Appendix I to find the logarithm of a number not shown in the "number" column. By recalling that the mantissa does not change when the decimal point moves, we may be able to determine the desired logarithm. For example, the number 450 does not appear in the number column of the table. However, the number 45 has the same mantissa as 450; the only difference between the two logs is in their characteristics. Thus the logarithm of 450 is 2.65321.

Practice problems. Find the logarithms of the following numbers:

1. 64	2. 98	3. 6400	4. 9.8

Answers:

1. 1.80618	2. 1.99123
3. 3.80618	4. 0.99123

THE SLIDE RULE

In 1620, not long after the invention of logarithms, Edmond Gunter showed how logarithmic calculations could be carried out mechanically. This is done by laying off lengths on a rule, representing the logarithms of numbers, and by combining these lengths in various ways. The idea was developed and with the contributions of Mannheim in 1851 the slide rule came into being as we know it today.

The slide rule is a mechanical device by which we can carry out any arithmetic calculation with the exception of addition and subtraction. The most common operations with the slide rule are multiplication, division, finding the square or cube of a number, and finding the square root or cube root of a number. Also trigonometric operations are frequently performed. The advantage of the slide rule is that it can be used with relative ease to solve complicated problems. One limitation is that it will give results with a maximum of only three accurate significant digits. This is sufficient in most calculations, however, since most physical constants are only correct to two or three significant digits. When greater accuracy is required, other methods must be used.

A simplified diagram of a slide rule is pictured in figure 8-1. The sliding, central part of the rule is called the SLIDE. The movable glass or plastic runner with a hairline imprinted on it is called the INDICATOR. There is a C scale printed on the slide, and a D scale exactly the same as the C scale printed on the BODY or STOCK of the slide rule. The mark that is associated with the primary number 1 on any slide rule scale is called the INDEX. There is

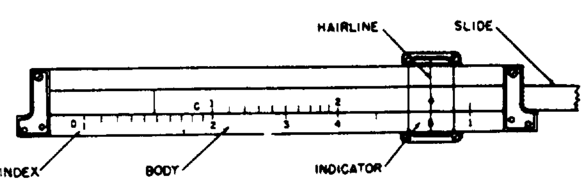

Figure 8-1.—Simplified diagram of a slide rule.

an index at the extreme left and at the extreme right on both the C and D scales. There are other scales, each having a particular use. Some of these will be mentioned later.

SLIDE RULE THEORY

We have mentioned that the slide rule is based on logarithms. Recall that, to multiply two numbers, we simply add their logarithms. Previously we found these logarithms in tables, but if the logarithms are laid off on scales such as the C and D scale of the slide rule, we can add the lengths, which represent these logarithms. To make such a scale we could mark off mantissas ranging from 0 to 1 on a rule as in figure 8-2. We then find in the tables the logarithms for numbers ranging from 1 to 10 and write the number opposite its corresponding logarithm on the scale.

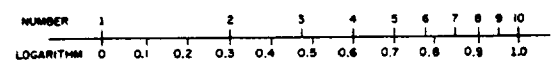

Figure 8-2.—Logarithms and corresponding numbers on a scale.

Table 8-7 lists the numbers 1 through 10 and their corresponding logarithms to three places. These numbers are written opposite their logarithms on the scale shown in figure 8-2. If we have two such scales, exactly alike, arranged so that one of them is free to slide along the other, we can perform the operation of multiplication, for example, by ADDING LENGTHS; that is, by adding logarithms. For example, if we wish to multiply 2 x 3, we find the logarithm of 2 on the stationary scale and move the sliding scale so that its index is over that mark. We then add the logarithm of 3 by finding that logarithm on the sliding scale and by reading below it, on the stationary scale, the logarithm that is the sum of the two.

Since we are not interested in the logarithms themselves, but rather in the numbers they represent, it is possible to remove the logarithmic notation on the scale in figure 8-2, and leave only the logarithmically spaced number scale. The C and D scales of the ordinary slide rule are made up in this manner. Figure 8-3 shows the multiplication of 2 x 3. Although the logarithm scales have been removed, the numbers 2 and 3 in reality signify the logarithms of

Table 8-7.—Numbers and their corresponding logarithms.

Number	Logarithm	Number	Logarithm
1	0.000	6	0.778
2	0.301	7	0.845
3	0.477	8	0.903
4	0.602	9	0.954
5	0.699	10	1.000

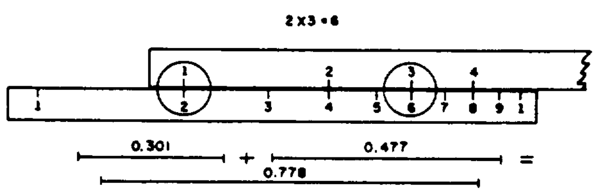

Figure 8-3.—Multiplication by use of the slide rule.

2 and 3, namely, 0.301 and 0.477; the product 6 on the scale really signifies the logarithm of 6, that is, 0.778. Thus, although logarithms are the underlying principle, we are able to work with the numbers directly.

It should be noted that the scale is made up from mantissas only. The characteristic must be determined separately as in the case where tables are used. Since mantissas identify only the digit sequence, the digit 3 on the slide rule represents not only 3 but 30, 300, 0.003, 0.3, and so forth. Thus, the divisions may represent the number multiplied or divided by any power of 10. This is true also for numbers that fall between the divisions. The digit sequence, 1001, could represent 100.1, 1.001, 0.01001, and so forth. The following example shows the use of the same set of mantissas which appear in the foregoing example, but with a different characteristic and, therefore, a different answer:

EXAMPLE: Use logs (positions on the slide rule) to multiply 20 times 30.

SOLUTION:

log 20 = 1.301 (2 on the slide rule)

log 30 = 1.477 (3 on the slide rule)

log of answer = 2.778 (6 on the slide rule)

Since the 2 in the log of the answer is merely the indicator of the position of the decimal point in the answer itself, we do not expect to find it on the slide rule scale. As in the foregoing example, we find the digit 6 opposite the multiplier 3. This time, however, the 6 represents 600, because the characteristic of the log represented by 6 in this problem is 2.

READING THE SCALES

Reading a slide rule is no more complicated than reading a yard stick or ruler, if the differences in its markings are understood.

Between the two indices of the C or D scales (the large digit 1 at the extreme left and right of the scales) are divisions numbered 2, 3, 4, 5, 6, 7, 8, and 9. Each length between two consecutive divisions is divided into 10 sections and each section is divided into spaces. (See fig. 8-4.)

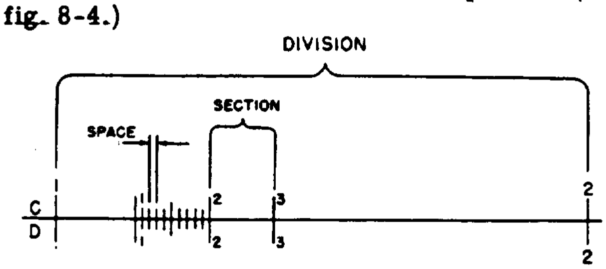

Figure 8-4.—Division, section, and space of a slide rule scale.

Notice that the division between 1 and 2 occupies about one-third of the length of the rule. This is sufficient space in which to write a number for each of the section marks. The sections in the remaining divisions are not numbered, because the space is more limited. Notice also that in the division between 1 and 2, the sections are each divided into 10 spaces. The sections of the divisions from 2 to 4 are subdivided into only 5 spaces, and those from 4 to the right index are subdivided into only 2 spaces. These subdivisions are so arranged because of the limits of space.

Only the sequence of significant digits is read on the slide rule. The position of the decimal point is determined separately. For example, if the hairline of the indicator is in the left-hand position shown in figure 8-5, the significant digits are read as follows:

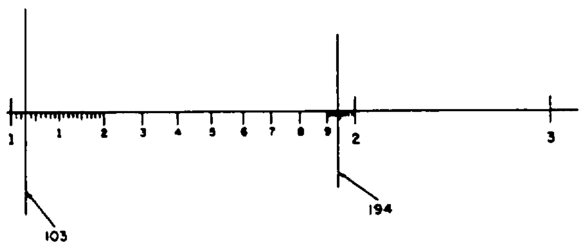

Figure 8-5.—Readings in the first division of a slide rule.

1. Any time the hairline falls in the first division, the first significant digit is 1.
2. Since the hairline lies between the index and the first section mark, we know the number lies between 1.0 and 1.1, or 10 and 11, or 100 and 110, etc. The second significant digit is 0.
3. We next find how far from the index the hairline is located. It lies on the marking for the third space.
4. The three significant digits are 103.

In the second example shown in figure 8-5, the hairline is located in the first division, the ninth section, and on the fourth space mark of that section. Therefore, the significant digits are 194.

Thus, we see that any number falling in the first division of the slide rule will always have 1 as its first significant digit. It can have any number from 0 through 9 as its second digit, and any number from 0 through 9 as its third digit. Sometimes a fourth digit can be roughly approximated in this first division, but the number is really accurate to only three significant digits.

In the second and third divisions, each section is divided into only 5 spaces. (See fig. 8-6.) Thus, each space is equal to 0.2 of the section. Suppose, for example, that the hairline lies on the third space mark after the large 2 indicating the second division. The first significant digit is 2. Since the hairline lies between 2 and the first section mark, the second digit is 0. The hairline lies on the third space mark or 0.6 of the way between the division mark and the first section mark, so the third digit is 6. Thus, the significant digits are 206. Notice that if the hairline lies on a space mark the third digit can be written accurately; otherwise it must be approximated.

From the fourth division to the right index, each section is divided into only two spaces. Thus, if the hairline is in the fourth division and lies on the space mark between the sixth and seventh sections, we would read 465. If the hairline did not fall on a space mark, the third digit would have to be approximated.

OPERATIONS WITH THE SLIDE RULE

There are two parts in solving problems with a slide rule. In the first part the slide rule is used to find the digit sequence of the final result. The second part is concerned with the placing of the decimal point in the result. Let us consider first the digit sequence in multiplication and division.

Multiplication

Multiplication is performed on the C and D scales of the slide rule. The following procedure is used:

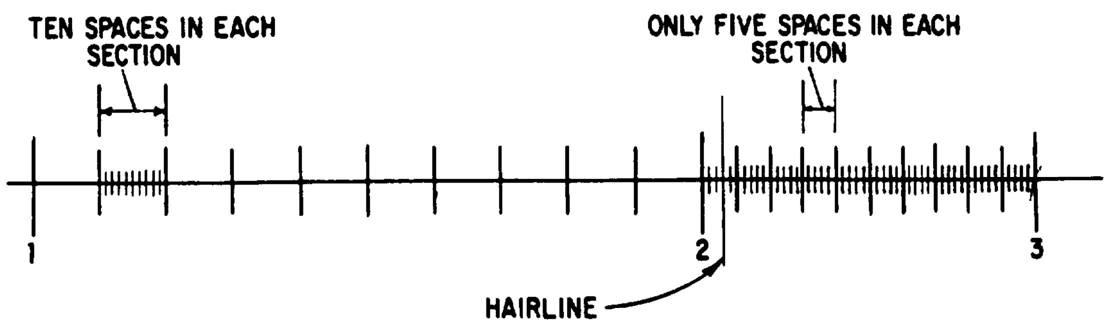

Figure 8-6.—Reading in the second division of a slide rule.

1. Locate one of the factors to be multiplied on the D scale, disregarding the decimal point.
2. Place the index of the C scale opposite that number.
3. Locate the other factor on the C scale and move the hairline of the indicator to cover this factor.
4. The product is on the D scale under the hairline.

Sometimes in multiplying numbers, such as 25 x 6, the number on the C scale extends to the right of the stock and the product cannot be read. In such a case, we simply shift indices. Instead of the left-hand index of the C scale, the right-hand index is placed opposite the factor on the D scale. The rest of the problem remains the same. By shifting indices, we are simply multiplying or dividing by 10, but this plays no part in reading the significant digits. Shifting indices affects the characteristic only.

EXAMPLE: 252 x 3 = 756

1. Place the left index of the C scale over 252.
2. Locate 3 on the C scale and set the hairline of the indicator over it.
3. Under the hairline on the D scale read the product, 756.

EXAMPLE: 4 x 64 = 256

1. Place the right index of the C scale over 4.
2. Locate 64 on the C scale and set the hairline of the indicator over it.
3. Under the hairline on the D scale read the product, 256.

Practice problems. Determine the following products by slide rule to three significant digits:

1. 2.8 x 16	3. 6 x 85
2. 7 x 1.3	4. 2.56 x 3.5

Answers:

1. 44.8	3. 510
2. 9.10	4. 8.96

Division

Division being the inverse of multiplication, the process of multiplication is reversed to perform division on a slide rule. We subtract the length representing the logarithm of the divisor from the length representing the logarithm of the dividend to get the logarithm of the quotient.

The procedure is as follows:

1. Locate the dividend on the D scale and place the hairline of the indicator over it.
2. Move the slide until the divisor (on the C scale) lies under the hairline.
3. Read the quotient on the D scale opposite the C scale index.

If the divisor is greater numerically than the dividend, the slide will extend to the left. If the divisor is less, the slide will extend to the right. In either case, the quotient is the number on the D scale that lies opposite the C scale index, falling within the limits of the D scale.

EXAMPLE: 6 ÷ 3 = 2

1. Locate 6 on the D scale and place the hairline of the indicator over it.
2. Move the slide until 3 on the C scale is under the hairline.
3. Opposite the left C scale index, read the quotient, 2, on the D scale.

EXAMPLE: 378 ÷ 63 = 6

1. Locate 378 on the D scale and move the hairline of the indicator over it.
2. Move the slide to the left until 63 on the C scale is under the hairline.
3. Opposite the right-hand index of the C scale, read the quotient, 6, on the D scale.

Practice problems. Determine the following quotients by slide rule.

1. 126 ÷ 3	3. 142 ÷ 71
2. 960 ÷ 15	4. 459 ÷ 17

Answers:

1. 42	3. 2
2. 64	4. 27

PLACING THE DECIMAL POINT

Various methods have been advanced regarding the placement of the decimal point in numbers derived from slide rule computations. Probably the most universal and most easily remembered method is that of approximation.

The method of approximation means simply the rounding off of numbers and the mechanical shifting of decimal points in the numbers of the problem so that the approximate size of the solution and the exact position of the decimal point will be seen from inspection. The slide rule may then be used to derive the correct sequence of significant digits. The method may best be demonstrated by a few examples. Remember, shifting the decimal point in a number one place to the left is the same as dividing by 10. Shifting it one place to the right is the same as multiplying by 10. Every shift must be compensated for in order for the solution to be correct.

EXAMPLE: 0.573×1.45

SOLUTION: No shifting of decimals is necessary here. We see that approximately 0.6 is to be multiplied by approximately 1 1/2. Immediately, we see that the solution is in the neighborhood of 0.9. By slide rule we find that the significant digit sequence of the product is 832. From our approximation we know that the decimal point is to the immediate left of the first significant digit, 8. Thus,

$0.573 \times 1.45 = 0.832$

EXAMPLE: 239×52.3

SOLUTION: For ease in multiplying, we shift the decimal point in 52.3 one place to the left, making it 5.23. To compensate, the decimal point is shifted to the right one place in the other factor. The new position of the decimal point is indicated by the presence of the caret symbol.

$$239.0_{\wedge} \times 5_{\wedge}2.3$$

Our problem is approximately the same as

$$2{,}400 \times 5 = 12{,}000$$

By slide rule the digit sequence is 125. Thus,

$$239 \times 52.3 = 12{,}500$$

EXAMPLE: 0.000134×0.092

SOLUTION:

Shifting decimal points, we have

$$0_{\wedge}00.000134 \times 0.09_{\wedge}2$$

Approximation: $9 \times 0.0000013 = 0.0000117$. By slide rule the digit sequence is 123. From approximation the decimal point is located as follows:

$$0.0000123$$

Thus,

$$0.000134 \times 0.092 = 0.0000123$$

EXAMPLE: $\dfrac{53.1}{42.4}$

SOLUTION: The decimal points are shifted so that the divisor becomes a number between 1 and 10. The method employed is cancellation.

Shifting decimal points, we have

$$\frac{5_{\wedge}3.1}{4_{\wedge}2.4}$$

Approximation: $\frac{5}{4} = 1.2$

Digit sequence by slide rule:

$$1255$$

Placing the decimal point from the approximation:

$$1.255$$

Thus,

$$\frac{53.1}{42.4} = 1.255$$

EXAMPLE: $\dfrac{0.00645}{0.0935}$

SOLUTION:

Shifting decimal points

$$\frac{0.00_{\wedge}645}{0.09_{\wedge}35}$$

Approximation:

$$\frac{0.6}{9} = 0.07$$

Digit sequence by slide rule: 690

Placing the decimal point from the approximation:

$$0.0690$$

Thus,

$$\frac{0.00645}{0.0935} = 0.0690$$

Practice problems. Solve the following problems with the slide rule and use the method of approximation to determine the position of the decimal point:

1. 0.00453 x 0.1645	3. 0.0362 x 1.21
2. 53.1 ÷ 1.255	4. 67 ÷ 316

Answers:

1. 0.000745	3. 0.0438
2. 42.4	4. 0.212

MULTIPLICATION AND DIVISION COMBINED

In problems such as

$$\frac{0.644 \times 330}{161 \times 12}$$

it is generally best to determine the position of the decimal point by means of the method of approximation and to determine the significant digit sequence from the slide rule. Such problems are usually solved by dividing and multiplying alternately throughout the problem. That is, we divide 0.644 by 161, multiply the quotient by 330, and divide that product by 12.

Shifting decimal points, we have

$$\frac{0_{\wedge}0.644 \times 3_{\wedge}30}{1_{\wedge}61 \times 1_{\wedge}2}$$

Since there is a combined shift of three places to the left in the divisor, there must also be a combined shift of three places to the left in the dividend.

$$\text{Approximation: } \frac{0.06 \times \overset{2}{\cancel{3}}}{\cancel{1}.\cancel{5}} = 0.06 \times 2 = 0.12$$

The step-by-step process of determining the significant digit sequence of this problem is as follows:

1. Place the hairline over 644 on the D scale.
2. Draw the slide so that 161 of the C scale lies under the hairline opposite 644.
3. Opposite the C scale index (on the D scale) is the quotient of 644 ÷ 161. This is to be multiplied by 330, but 330 projects beyond the rule so the C scale indices must be shifted.
4. After shifting the indices, find 330 on the C scale and place the hairline over it. Opposite 330 under the hairline on the D scale is the product of $\frac{644}{161} \times 330$.
5. Next, move the C scale until 12 is under the hairline. Opposite the C scale index (on the D scale) is the final quotient. The digit sequence is 110.

The decimal point is then placed according to our approximation: 0.11. Thus,

$$\frac{0.644 \times 330}{161 \times 12} = 0.11$$

Practice problems. Solve the following problems, using a slide rule:

1. $\frac{22 \times 78.5 \times 157}{17 \times 18.3 \times 85}$

2. $\frac{432 \times 9{,}600}{25{,}600 \times 198}$

3. $\frac{2.77 \times 0.064}{0.17 \times 1.97}$

Answers:

1. 10.2	2. 0.817	3. 0.529

SQUARES

Squares of numbers are found by reference to the A scale. The numbers on the A scale are the squares of those on the D Scale. The A scale is really a double scale, each division being one-half as large as the corresponding division on the D scale. The use of a double scale for squaring is based upon the fact that the logarithm of the square of a number is twice as large as the logarithm of the number itself. In other words,

$$\log N^2 = 2 \log N$$

This is reasonable, since

$$\begin{aligned} \log N^2 &= \log (N \times N) \\ &= \log N + \log N \end{aligned}$$

For a numerical example, suppose that we seek to square 2 by means of logarithms.

$$\begin{aligned}\log 2 &= 0.301\\ \log 2^2 &= 2 \log 2\\ &= 2 \times 0.301\\ &= 0.602\end{aligned}$$

Since each part of the A scale is half as large as the corresponding part of the D scale, the logarithm 0.602 on the A scale will be the same length as the logarithm 0.301 on the D scale. That is, these logarithms will be opposite on the A and D scales. On the A scale as on the D scale, the numbers are written rather than their logarithms. Select several numbers on the D scale, such as 2, 4, 8, 11, and read their squares on the A scale, namely 4, 16, 64, 121.

Notice also that the same relation exists for the B and C scales as for the A and D scales. Of interest, also, is the fact that since the A and B scales are made up as are the C and D scales, they too could be used for multiplying or dividing.

Placing the Decimal Point

Usually the decimal may be placed by the method of approximation. However, close observation will reveal certain facts that eliminate the need for approximations in squaring numbers. Two rules suffice for squaring whole or mixed numbers, as follows:

1. When the square of a number is read on the left half of the A scale, that number will contain twice the number of digits to the left of the decimal point in the original number, less 1.
2. When the square of a number is read on the right half of the A scale, that number will contain twice the number of digits to the left of the decimal point in the original number.

EXAMPLE: Square 2.5.

SOLUTION: Place the hairline over 25 on the D scale. Read the digit sequence, 625, under the hairline in the left half of the A scale.

By rule 1: (2 x number of digits) - 1 = 2(1) - 1 = 1. There is one digit to the left of the decimal point. Thus,

$$(2.5)^2 = 6.25$$

EXAMPLE: Square 6,340.

SOLUTION:

Digit sequence, right half A scale: 402.

By rule 2: 2 x number of digits = 2 x 4 = 8 (digits in answer). Thus,

$$(6{,}340)^2 = 40{,}200{,}000$$

Positive Numbers Less Than One

If positive numbers less than one are to be squared, a slightly different version of the preceding rules must be employed. Count the zeros between the decimal point and the first nonzero digit. Consider this count negative. Then the number of zeros between the decimal point and the first significant digit of the squared number may be found as follows:

1. Left half A scale: Multiply the zeros counted by 2 and subtract 1.
2. Right half A scale: Multiply the zeros counted by 2.

EXAMPLE: Square 0.0045

SOLUTION:

Digit sequence, right half A scale: 2025.

By rule 2: 2(-2) = -4. (Thus, 4 zeros between the decimal point and the first digit.)

$$(0.0045)^2 = 0.00002025$$

EXAMPLE: Square 0.0215

SOLUTION:

Digit sequence, left half A scale: 462.

By rule 1: 2(-1) -1 = -3

$$(0.0215)^2 = 0.000462$$

SQUARE ROOTS

Taking the square root of a number with the slide rule is the inverse process of squaring a number. We find the number on the A scale, set the hairline of the indicator over it, and read the square root on the D scale under the hairline.

Positioning Numbers on the A Scale

Since there are two parts of the A scale exactly alike and the digit sequence could be found on either part, a question arises as to which section to use. Generally, we think of the left half of the rule as being numbered from 1 to 10 and the right half as being numbered from 10 to 100. The numbering continues– left half 100 to 1,000, right half 1,000 to 10,000, and so forth.

A simple process provides a check of the location of the number from which the root is to be taken. For whole or mixed numbers, begin at the decimal point of the number and mark off the digits to the left (including end zeros) in groups of two. This is illustrated in the following two examples:

1. $\sqrt{40{,}300.21}$
   $\sqrt{4'03'00.21}$
2. $\sqrt{2{,}034.1}$
   $\sqrt{20'34.1}$

Look at the left-hand group. If it is a 1-digit number, use the left half of the A scale. If it is a 2-digit number, use the right half of the A scale. The number in example 1 is thus located in the left half of the A scale and the number in example 2 is located in the right half.

Numbers Less Than One

For positive numbers less than one, begin at the decimal point and mark off groups of two to the right. This is illustrated as follows:

1. $\sqrt{0.000245}$
   $\sqrt{0.00'02'45}$
2. $\sqrt{0.00402}$
   $\sqrt{0.00'40'2}$

Looking from left to right, locate the first group that contains a digit other than zero. If the first figure in this group is zero, locate the number in the left half of the A scale. If the first figure is other than zero, locate the number in the right half of the A scale. Thus,

$\sqrt{0.00'02'45}$ is located left

and

$\sqrt{0.00'40'2}$ is located right

Powers of 10

When the square root of 10, 1,000, 100,000, and so forth, is desired, the center index is used. That is, when the number of digits in a power of 10 is even, use the center index.

The slide rule uses only the first three significant digits of a number. Thus, if the rule is used, $\sqrt{23451.6}$ must be considered as $\sqrt{23500.0}$. Likewise, 1.43567 would be considered 1.43000, and so forth. For greater accuracy, other methods must be used.

Practice problems. State which half of the A scale should be used for each of the following:

1. $\sqrt{432}$	5. $\sqrt{4{,}320}$
2. $\sqrt{0.014}$	6. $\sqrt{0.00301}$
3. $\sqrt{241.67}$	7. $\sqrt{0.0640}$
4. $\sqrt{0.00045}$	8. $\sqrt{9.41}$

Answers:

1. Left	5. Right
2. Left	6. Right
3. Left	7. Left
4. Left	8. Left

Placing the Decimal Point

To place the decimal point in the square root of a number, mark off the original number in groups of two as explained previously.

For whole or mixed numbers, the number of groups marked off is the number of digits including end zeros to the left of the decimal point in the root. The following problems illustrate this:

1. $\sqrt{23{,}415}$
   $\sqrt{2'34'15}$ Three digits to left of decimal point in square root
2. $\sqrt{421{,}562.4}$
   $\sqrt{42'15'62.4}$ Three digits to left of decimal point in square root
3. $\sqrt{231.321}$
   $\sqrt{2'31.321}$ Two digits to left of decimal point in square root

For positive numbers less than one, there will be one zero in the square root between the decimal point and the first significant digit for every pair of zeros counted between the decimal point and the first significant digit of the original number. This is illustrated as follows:

1. $\sqrt{0.0004}$

   $\sqrt{0.00'04}$ One zero before first digit in square root

2. $\sqrt{0.00008}$

   $\sqrt{0.00'00'8}$ Two zeros before first digit in square root

3. $\sqrt{0.08'}$ No zeros before first digit in square root

EXAMPLE: $\sqrt{4,521}$

$\sqrt{45'21}$

(Two digits in left-hand group)

Place the hairline over 452 on the right half of the A scale. Read the digit sequence of the root, 672, on the D scale under the hairline. Since there are two groups in the original number, there are two digits to the left of the decimal point in the root. Thus,

$$\sqrt{4,521} = 67.2$$

EXAMPLE: $\sqrt{0.000741}$

$\sqrt{0.00'07'41}$

(First figure is zero in this group)

Place the hairline over 741 on the left half of the A scale. Read the digit sequence of the root, 272, under the hairline on the D scale. Since there is one pair of zeros to the left of the group containing the first digit, there is one zero between the decimal point and the first significant digit of the root. Thus,

$$\sqrt{0.000741} = 0.0272$$

Practice problems. Evaluate each of the following by means of a slide rule:

1. $(17.75)^2$
2. $(0.65)^2$
3. $\sqrt{9.42}$
4. $\sqrt{0.074}$

Answers:

1. 315
2. 0.422
3. 3.07
4. 0.272

CUBES AND CUBE ROOTS

Cubes and cube roots are read on the K and D scales of the slide rule. On the K scale are compressed three complete logarithmic scales in the same space as that of the D scale. Thus, any logarithm on the K scale is three times the logarithm opposite it on the D scale. To cube a number by logarithms, we multiply its logarithm by three. Therefore, the logarithms of cubed numbers will lie on the K scale opposite the numbers on the D scale.

As with the other slide rule scales mentioned, the numbers the logarithms represent, rather than the logarithmic notations, are printed on the rule. In the left-hand third of the K scale, the numbers range from 1 to 10; in the middle third they range from 10 to 100; and in the right-hand third, they range from 100 to 1,000.

To cube a number, find the number on the D scale, place the hairline over it, and read the digit sequence of the cubed number on the K scale under the hairline.

Placing the Decimal Point

The decimal point of a cubed whole or mixed number may be easily placed by application of the following rules:

1. If the cubed number is located in the left third of the K scale, its number of digits to the left of the decimal point is 3 times the number of digits to the left of the decimal point in the original number, less 2.
2. If the cubed number is located in the middle third of the K scale, its number of digits is 3 times the number of digits of the original number, less 1.
3. If the cubed number is located in the right third of the K scale, its number of digits is 3 times the number of digits of the original number.

EXAMPLE: $(1.6)^3$

SOLUTION: Place the hairline over 16 on D scale. Read the digit sequence, 409, on the K scale under the hairline.

Number of digits to left of decimal point in the number 1.6 is 1 and the cubed number is in the left-hand third of the K scale.

$$3 \times (\text{No. of digits})-2 = (3 \times 1)-2 = 1$$

Therefore,

$$(1.6)^3 = 4.09$$

EXAMPLE: $(4.1)^3$

Digit sequence = 689.

SOLUTION: Number of digits to left of decimal point in the number 4.1 is 1, and the cubed number is in the middle third of the K scale.

$$3 \times (\text{No. of digits})-1 = (3 \times 1)-1 = 2$$

Therefore,

$$(4.1)^3 = 68.9$$

EXAMPLE: $(52)^3$

SOLUTION: Digit sequence = 141.

Number of digits to left of decimal point in the number 52 is 2, and the cubed number is in the right-hand third of the K scale.

$$3 \times \text{No. of digits} = 3 \times 2 = 6$$

Therefore,

$$(52)^3 = 141{,}000$$

Positive Numbers Less Than One

If positive numbers less than one are to be cubed, count the zeros between the decimal point and the first nonzero digit. Consider the count negative. Then the number of zeros between the decimal point and the first significant digit of the cubed number may be found as follows:

1. Left third of K scale: Multiply the zeros counted by 3 and subtract 2.
2. Middle third of K scale: Multiply the zeros counted by 3 and subtract 1.
3. Right third of K scale: Multiply the zeros counted by 3.

EXAMPLE: Cube 0.034

SOLUTION: Digit sequence = 393

Zero count of 0.034 = -1, and 393 is in the middle third of the K scale.

$$3 \times (\text{No. of zeros})-1 = (3 \times -1)-1 = -4$$

Therefore,

$$(0.034)^3 = 0.0000393$$

Practice problems. Cube the following numbers using the slide rule.

1. 21 2. 0.7 3. 0.0128 4. 404

Answers:

1. 9260
2. 0.342
3. 0.0000021
4. 66,000,000

Cube Roots

Taking the cube root of a number on the slide rule is the inverse process of cubing a number. To take the cube root of a number, find the number on the K scale, set the hairline over it, and read the cube root on the D scale under the hairline.

POSITIONING NUMBERS ON THE K SCALE.—Since a given number can be located in three positions on the K scale, the question arises as to which third of the K scale to use when locating a number. Generally, the left index, the left middle index, the right middle index, and the right index are considered to be numbered as shown in figure 8-7.

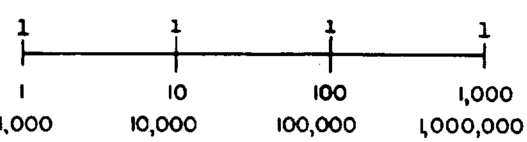

Figure 8-7.—Powers of 10 associated with K-scale indices.

A system similar to that used with square roots may be used to locate the position of a number on the K scale. Groups of three are used rather than groups of two. The grouping for cube root is illustrated as follows:

1. $\sqrt[3]{40,531.6}$

   $\sqrt[3]{40'531.6}$

2. $\sqrt[3]{4,561.43}$

   $\sqrt[3]{4'561.43}$

3. $\sqrt[3]{0.000043}$

   $\sqrt[3]{0.000'043}$

For whole or mixed numbers the following rules apply:

1. If the left-hand group contains one digit, locate the number in the left third of the K scale.
2. If the left group contains two digits, locate the number in the middle third of the K scale.
3. If the left group contains three digits, locate the number in the right third of the K scale.

The following examples illustrate the foregoing rules:

1. $\sqrt[3]{4'561.43}$

   (One digit)—left third K scale.

2. $\sqrt[3]{40'531.6}$

   (Two digits)—middle third K scale.

3. $\sqrt[3]{453'361}$

   (Three digits)—right third of K scale.

For positive numbers less than one, look from left to right and find the first group that contains a digit other than zero.

1. If the first two figures in this group are zeros, locate the number in the left third of the K scale.
2. If only the first figure in this group is zero, locate the number in the middle third of the K scale.
3. If the first figure of the group is not zero, locate the number in the right third of the K scale.

The following examples illustrate these rules:

1. $\sqrt[3]{0.000'004'53}$

   (Two zeros)—left third K scale.

2. $\sqrt[3]{0.000'050'43}$

   (One zero)—middle third K scale.

3. $\sqrt[3]{0.000'000'430}$

   (No zero)—right third K scale.

PLACING THE DECIMAL POINT.—To place the decimal point in the cube root of a number, we use the system of marking off in groups of three as shown above.

For whole or mixed numbers, there is one digit in the root to the left of the decimal point for every group marked in the original number. Thus,

$$\sqrt[3]{4'531.6}$$

(Two digits in root to left of decimal point.)

For positive numbers less than one, there will be one zero in the root between the decimal point and the first significant digit for every three zeros counted between the decimal point and the first significant digit of the original number. Thus,

$$\sqrt[3]{0.000'000'004}$$

(Two zeros between decimal point and first significant digit of root.)

EXAMPLE: $\sqrt[3]{216000.4}$

$\sqrt[3]{216'000.4}$

(Three digits in left group)

Place the hairline over 216 in the right third of the K scale. Read the digit sequence, 6, under the hairline on the D scale. Since there are two groups in the original number, there are two digits to the left of the decimal point in the root. Thus,

$$\sqrt[3]{216000.4} = 60$$

EXAMPLE: $\sqrt[3]{0.0000451}$

$\sqrt[3]{0.000'045'1}$

(Only first figure is zero in this group)

Place the hairline over 451 in the middle third of the K scale. Read the digit sequence, 357, under the hairline on the D scale. Since there is one group of three zeros, there is one zero between the decimal point and the first significant digit of the root. Thus,

$$\sqrt[3]{0.0000451} = 0.0357$$

POWERS OF 10.—To take the cube root of a power of 10, mark it off as explained in the preceding paragraphs. The number in the left group will then be 1, 10, or 100. We know that the cube root of 10 is a number between 2 and 3. Thus, for the cube root of any number whose left group is 10, use the K scale index which lies between 2 and 3 on the D scale. The cube root of 100 lies between 4 and 5. Therefore, for a number whose left group is 100, use the K scale index between 4 and 5 on the D scale.

Practice problems. Following are some problems and the digit sequence (d. s.) of the roots. Locate the decimal point for each root.

1. $\sqrt[3]{0.000023}$ d. s. 2844
2. $\sqrt[3]{0.051}$ d. s. 371
3. $\sqrt[3]{127}$ d. s. 5026
4. $\sqrt[3]{204{,}000}$ d. s. 589
5. $\sqrt[3]{734{,}000{,}000}$ d. s. 902
6. $\sqrt[3]{4{,}913}$ d. s. 17

Answers:

1. 0.02844
2. 0.371
3. 5.026
4. 58.9
5. 902
6. 17